1. **Arithmetic Sequences** : It's a number pattern that keeps adding the same number every time.

Imagine you get 2 slices of pizza on day one, and then every day after that, your parents you 3 more slices. 🍕🍕🍕

Your pizza stash grows like this :

2, 5 , 8, 11, 14...

Each day the number goes up by the same amount.

Key word: Arithmetic = Add the same thing every time

To find any term in the sequences, use this :

$$\boxed{a_n = a_1 + (n-1) \times d}$$

a_n = the term you want

n = the position

d = the number you keep adding

Example : You earn 10 coins for your first quest, and every quest after that gives 4 more coins.

How many coins after 5 quests?

$$a_5 = a_1 + (n-1) \times 4$$

$$a_5 = 10 + (5-1) \times 4$$

$$a_5 = 10 + 16 = 26$$

After 5 quests, you have 26 coins.

2. **Geometric Sequences**: It's a number pattern where the numbers multiply each time instead of adding. If arithmetic is "step by step", then geometric is "zoom zoom!" - they grow fast.

Imagine you bake cookies. And, every hour, you double what you had before; 1,2,4,8,16,32 ...

You're multiplying by 2 each time.

That "keep multiplying" pattern = geometric sequence

$$\boxed{a_n = a_1 \times r^{(n-1)}}$$

a_n = the term you want

a_1 = the first number

r = the number you are multiplying each time

n = position

Example: You invest 3 dollars and it triples every month. The sequence looks like 3, 9, 27 , 81 ...

How much is your investment after 5 months?

$$a_5 = a_1 \times r^{(n-1)}$$

$$a_5 = 3 \times 3^{(5-1)}$$

$$a_5 = 3 \times 3^4$$

$$a_5 = 3 \times 81 = 243$$

After 5 months, you have 243 dollars.

3. **Recursive and Explicit Formulas**

A. Recursive : You build the sequence step-by-step. Each term depends on the one before it.

You must be told the starting number.

Example 1 : You start with 2 cookies, and you get 3 more cookies each day.

Day 1: 2

Day 2: 2 + 3 = 5

Day 3: 5 + 3 = 8

...

Recursive formula for the above example:

$a_1 = 2$

$a_n = a_{n-1} + 3$

To get today's cookies, add 3 to yesterday's!

Generic recursive formula for arithmetic sequence:

a_1 = first term

a_n = the term you want

a_{n-1} = the term before it

d = the common difference (what you add each time)

$$\boxed{a_n = a_{n-1} + d}$$

Example2 : You have 1 donut. Every day, your donut count triples (magical donuts!)

Day 1: $a_1 = 1$

Day 2: $a_2 = 1 \times 3 = 3$

Day 3: $a_3 = 3 \times 3 = 9$

Day 4: $a_4 = 9 \times 3 = 27$

...

Recursive formula for the above example:

$a_1 = 1$ Start with one donut

$a_n = a_{n-1} \times 3$ Multiply 3 each day to get next number

Generic recursive formula for geometric sequence:

$a_1 =$ starting number

$a_n =$ the term you want

$a_{n-1} =$ the term before it

$d =$ the multiplier (common ratio)

$\boxed{a_n = a_{n-1} \times d}$

$a_4 = a_3 \times d$ from the above example

$a_4 = 9 \times 3 = 27$

$a_5 = a_4 \times 3$

$a_5 = 27 \times 3 = 81$

B. Explicit formula : An explicit formula lets you find any term directly by using the position number.

Arithmetic sequence : $a_n = a_1 + (n-1) \times d$

Geometric sequence : $a_n = a_{n-1} \times d$

Example 1: You earn 4 points on Day 1. You get 2 more points every day.

This is an arithmetic sequence. 4,6,8,10,12...

$\boxed{\text{explicit formula : } a_n = a_1 + (n-1) \times d}$

n is the position , $a_1 = 4$, d = common difference

$a_5 = 4 + (5-1) \times d$

$a_5 = 4 + (4) \times 2$

$a_5 = 4 + 8 = 12$

Example 2: You start with 5 dollars, and every week, your money doubles.

This is a geometric sequence. 5,10, 20,40,80...

$\boxed{\text{explicit formula : } a_n = a_1 \times r^{(n-1)}}$

$a_1 = 5$, $r = 2$ (common ratio)

$a_n = 5 \times 2^{n-1}$

To find 6^{th} term, the dollars after 6weeks

$a_6 = 5 \times 2^{6-1} = 5 \times 64 = 320\ \$$