1. **Identifying functions from tables, graphs, and mappings**

A function is a rule where each input (x) gives you exactly one output (y).

Think : You type in a number => you get only one answer.

If an input gives two answers, it's not a function.

A. **From a table :** Check if any x value repeats with different y values.

Function table : Each input has only one output = Function

x	y
1	3
2	4
3	5

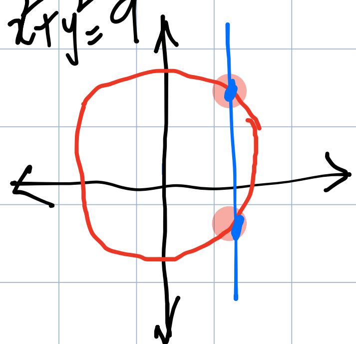
Not a function; same input, two outputs = Not a function

x	y
1	3
1	5
2	6

B. **From a Graph :** Draw a vertical line, if it touches the graph more than once, it's not a function.

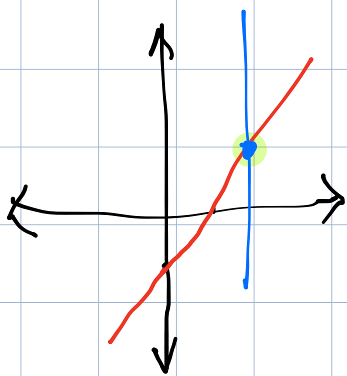
Not a function, vertical blue line has 2 intersections.

$x^2+y^2=9$



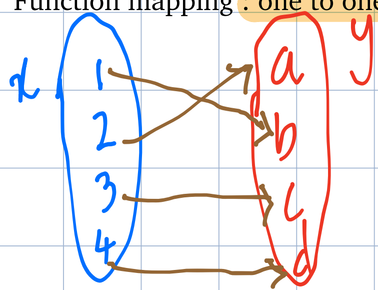
Is a function, vertical blue line has one intersection

$y = x + 1$

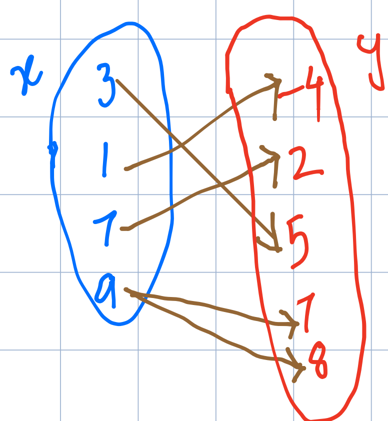


C. **From a mapping diagram** :

Function mapping : one to one



Not a function : 9 maps to 7 and 8.

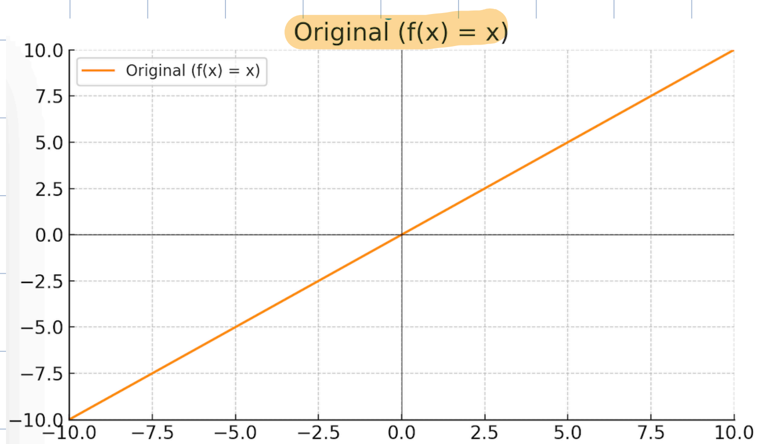


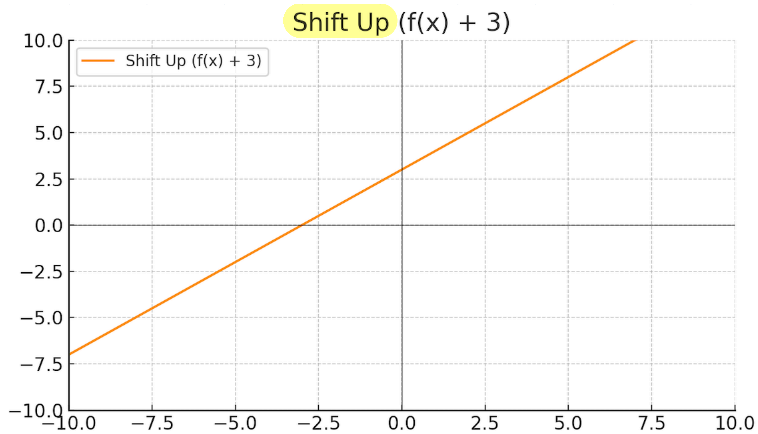
2. Function transformations

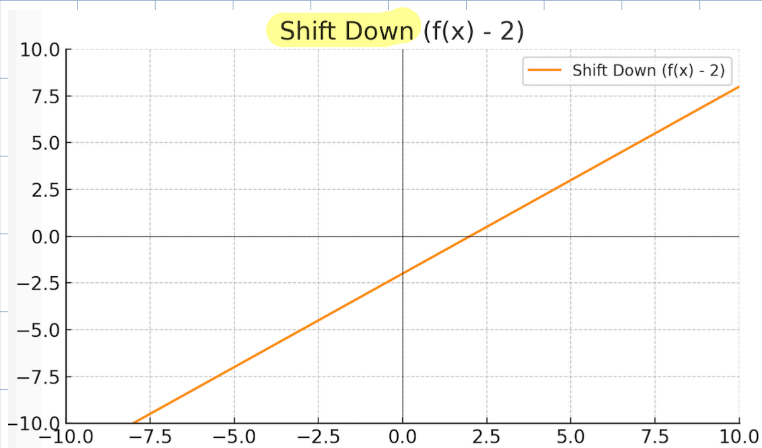
Function transformations change how a graph looks- by moving it, flipping it, or by stretching it.

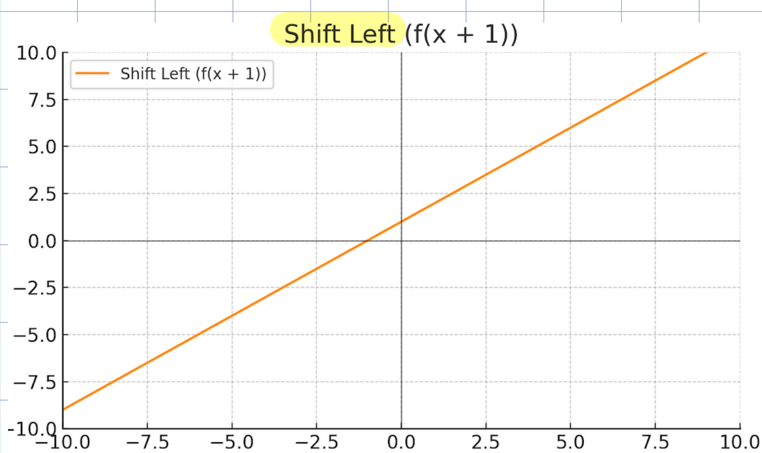
Think of editing a photo: Shift = move it, Reflect = flip it , Stretch = zoom it

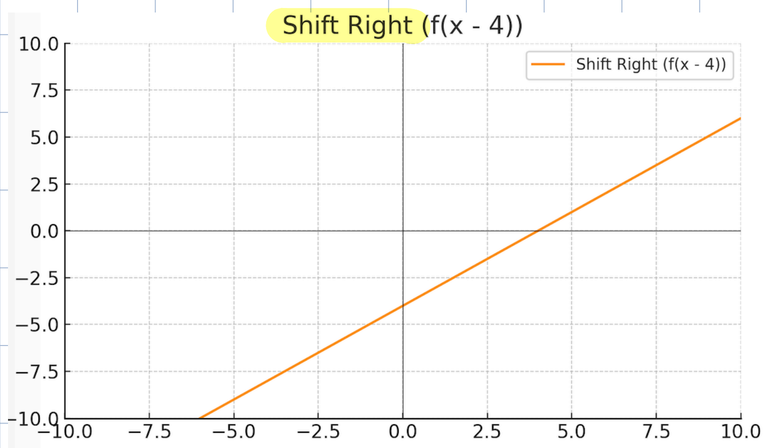
Type	What It Does	Example (based on f(x))
Shift up	Moves the graph higher	$f(x) + 3$
Shift down	Moves the graph lower	$f(x) - 2$
Shift left	Moves it to the left	$f(x + 1)$
Shift right	Moves it to the right	$f(x - 4)$
Reflect over x-axis	Flips the graph upside down	$-f(x)$
Stretch vertically	Makes it taller	$2f(x)$
Shrink vertically	Makes it flatter	$0.5f(x)$

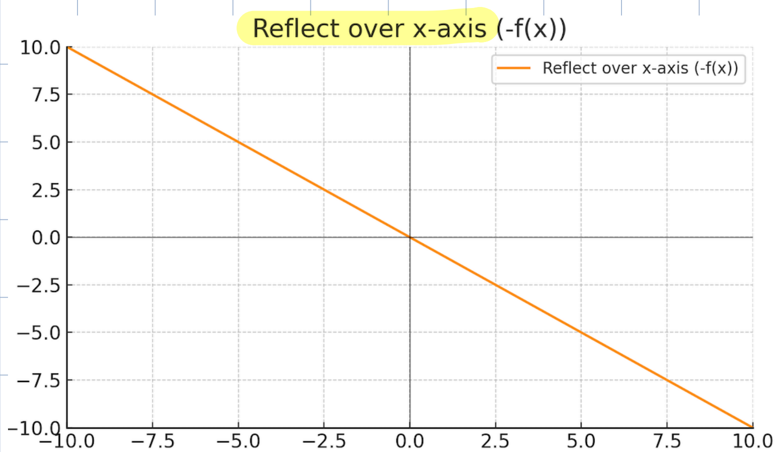


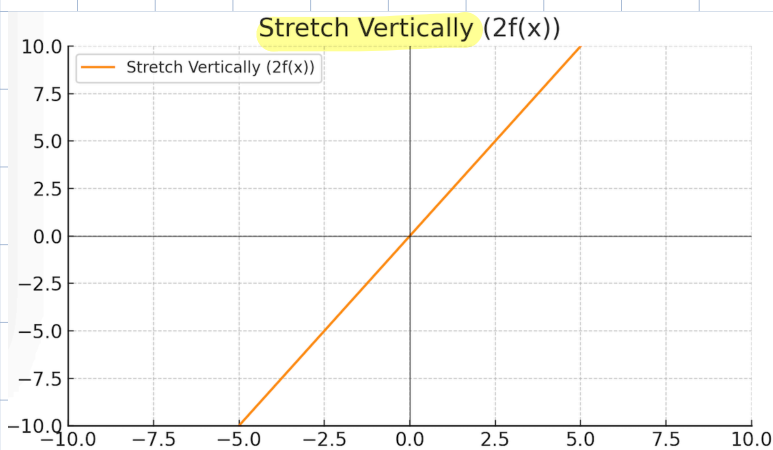

Shift Up (f(x) + 3)
Shift Up (f(x) + 3)
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0

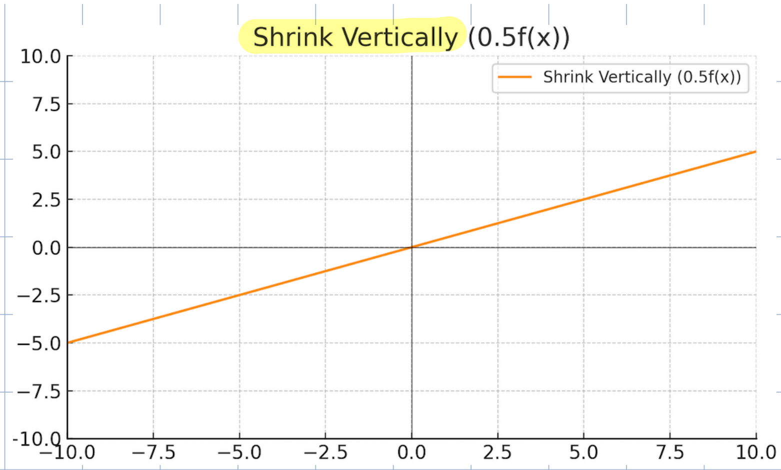

Shift Down (f(x) - 2)
Shift Down (f(x) - 2)
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0


Shift Left (f(x + 1))
Shift Left (f(x + 1))
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0


Shift Right (f(x - 4))
Shift Right (f(x - 4))
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0


Reflect over x-axis (-f(x))
Reflect over x-axis (-f(x))
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0


Stretch Vertically (2f(x))
Stretch Vertically (2f(x))
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−10.0
−10.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5
10.0



3. **Inverse relations**

An inverse relation is like undoing what a function does.

A function takes you forward, the inverse takes you backward.

Let's say you send $5 per hour at an arcade.

function $\Rightarrow$ $f(x) = 5x$

x = hours , $f(x)$ = money spent

Inverse $\Rightarrow$ $f^{-1}(x) = x \div 5$

x = money , $f^{-1}(x)$ = hours played

Steps to find inverse:

Step 1: Replace f(x) with y

Step 2: Switch x and y

Step 3: Solve for the new y

Example:

$f(x) = 2x + 3$

Step1 : $y = 2x+3$

Step2 : $x = 2y+3$

Step3: $x-3 = 2y \Rightarrow y = \frac{x-3}{2}$

So: $f^{-1}(x) = \frac{x-3}{2}$

4. **Piecewise functions**

A piecewise function is like a rulebook with sections. Each section has a different rule, depending on the input(x). It is made of pieces.

Real world example : Pizza delivery fee

Distance (miles)	**Delivery fee**
0 - 3	**$5**
4 - 6	**$7**
7+	**$10**

The fee changes based on how far away you live. That's what a piecewise function does.

Example function

$$f(x) = \begin{cases} x+2 & \text{if } x<0 \\ 3x & \text{if } x \geq 0 \end{cases}$$

$f(-2) = -2+2 = 0$ (use the first rule)

$f(1) = 3(1) = 3$ (use the second rule)

Imagine a video game with different levels of difficulty, Easy mode for levels 1-3, Medium for 4-6. Hard for 7+. Piecewise functions work the same way- different rule for different zone.

5. **Step functions**

A step function looks like a staircase on a graph. It jumps from one value to the next without connecting smoothly.

Think : Same value for a while, then a sudden jump!

Real life example: Bus fare

Miles Traveled	**Cost**
0-4 miles	$2
5-9 miles	$3
10+ miles	$4

If you go 4.1 miles, you pay $3 - no in-between price!

Example : Round down function

$f(x) = \lfloor x \rfloor$ $\Rightarrow$ round down to the nearest whole number

$f(2.9) = 2$

$f(5.4) = 5$

$f(-1.2) = -2$

Each time, it steps down to the closest whole number below it.

You're going up stairs, you stay on one step until it's time to jump to the next. There's no sliding.

6. **Domain and Range (In Context and Abstract)**

A. Domain and Range in Context (Real Life)

Domain : what you're allowed to put in (inputs).

Range : what you get out (outputs)

Example: You sell 1 to 5 cones per day, and each cone costs $3.

$f(x) = 3x$; x = number of cones sold

Domain = $\{1, 2, 3, 4, 5\}$

Range = $\{f(1), f(2), f(3), f(4), f(5)\}$

$= \{3, 6, 9, 12, 15\}$

B. Domain and Range in Abstract Math

Example: $f(x) = x^2$

You can plug in any real number -> no limits

Squaring never gives a negative number.

Domain ⇒ All real numbers

Domain = $(-\infty, \infty)$

Range ⇒ All numbers greater than equal to 0

Range = $[0, \infty)$

The functions works everywhere, but only gives positive results.

Concept	**Meaning**
Domain	What you can plug in (input x)
Range	What comes out (output f(x))
In Context	Inputs/Outputs that make sense in real life
Abstract	Inputs/Outputs based on math rules