Why Do We Need to Solve Equations?

Can be applied to real-world problems:

You're saving up for your favorite video game. You already have some money, and your parents agree to give you a little more each week. How many weeks until you can buy the game?

That's not just a money problem—It's an equation!

Solving One-Step Equations:

One step equations usually includes a variable (like X)—the unknown we're trying to find. These are equations you can solve with one step — either adding, subtracting, multiplying, or dividing.

Example 1: X+7 = 10

Step: Subtract 7 from both sides, X = 3

Example 2: Z x 5 = 25

Step: Divide both sides by 5 , Z = 5

Multi-Step Equations:

Multi-step equations require more than one step to solve. You might need to distribute, combine like terms, and inverse operations (add/subtract, multiply/divide).

Example: 3X + 4 = 16

Step 1: Subtract by 4, 3X = 12

Step 2: Divide by 3, X = 4

Example 2: 2(Z-3) = 10

Step 1: Distribute the 2, 2Z-6 = 10

Step 2: Add 6, 2Z = 16

Step 3: Divide by 2, Z = 8

Solving Equations with Variables on Both Sides:

An equation with variables on both sides means both sides have an unknown (like x). To solve, get all the x's on one side and the numbers on the other.

Example 1 (Real-world):

You and your friend are both collecting candies

You: 5 candies + 2 each day

Friend: 1 candy + 4 each day

One day, you'll both have the same amount.

An equation with variables on both sides helps you figure out when that happens.

Assume Z is the number of days the candies are being collected.

You collect: 5 candies + (2 x Z)

Your friend collects: 1 candy + (4 x Z)

Both have the same number of candies at the end.

$5 + (2 \times Z) = 1 + (4 \times Z)$

Step 1: Subtract 1 from both sides, $4 + 2Z = 4Z$

Step 2: Subtract 2Z from both sides, $4 = 2Z$

Step 3: Divide by 2, $Z = 2$.

You'll both have same number of candies after 2 days!

Example 2: $3X + 6 = 2X + 10$

Step 1: Subtract 2X from both sides, $X + 6 = 10$

Step 2: Subtract 6 from both sides, $X = 4$

Both sides are equal when $X = 4$.

Solving and Graphing Inequalities:

Inequalities are like equations, but instead of saying two things are exactly equal, you're saying one is greater, less, or not equal.

Real-world Problem: I need at least $10 to buy lunch–it's any number greater than or equal to 10. They help us describe a range of possible answers, not just one.

< means “less than”

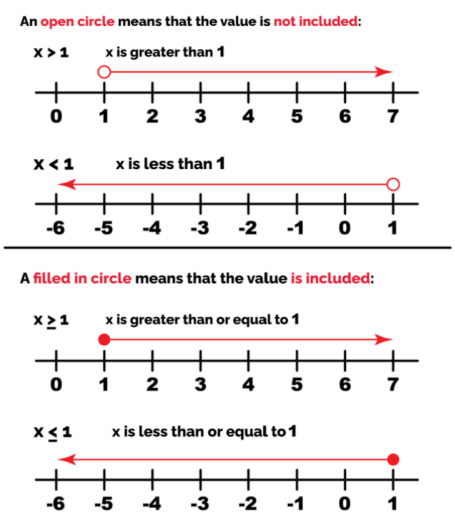
> means “greater than”

≤ means “less than or equal to”

≥ means “greater than or equal to”

Solving works just like equations — use inverse operations to isolate x.

One special rule: If you multiply or divide by a negative, you must flip the sign.



Example 1: Y + 3 < 10

Step: Subtract 3, Y < 7

Example 2: -2X > 6

Step: Divide by -2, and flip the sign, X < -3

Compound Inequalities:

Example: You’re allowed to be in the school cafeteria only if it’s after 11:30 AM but before 1:00 PM.

That’s a time range — and that’s what compound inequalities do They show when something is between two values, or outside a certain range.

Inequalities on a Number line:

Solving AND/Intersection Compound Inequalities

$3X + 1 < 4$ AND $-2X + 2 \leq 8$

$3X < 3$ AND $-2X \leq 6$

$X < 1$ AND $X \geq -3$ (Flip the sign, divided by -2)

$-3 \leq X < 1$

Absolute Value Equations and Inequalities:

The absolute value of a number means how far it is from 0 on the number line, no matter the direction.

It is always positive or 0.

Examples :

$|5| = 5 \Rightarrow$ 5 is 5 units from 0.

$|-5| = 5 \Rightarrow$ -5 is also 5 units from 0

$|0| = 0 \Rightarrow$ 0 is 0 units from 0.

Absolute values removes the negative sign, because it's only about distance, not direction.

Distance Between Locations:

Example: You live at house number 12. Your friend lives at house number x. You want to know how far apart you live:

Inequality setup $|X - 12| = 5$

Case 1 : $X - 12 = -5$, $X = 7$

Case 2: $X - 12 = 5$, $X = 17$

$X = 17$ or $X = 7$

Your friend could live at house 7 or house 17. Both are exactly 5 units away from house 12.

Grading Curves:

Example:

A teacher wants to give bonus points to anyone scoring within 5 points of 90.

Inequality setup : $|X-90| \leq 5$, This means distance from 90 must be 5 or less

Step 1: Remove the absolute value, write as a compound inequality

$-5 \leq X - 90 \leq 5$

Step 2: Solve, add 90 to all parts

$85 \leq X \leq 95$

Students who score between 85 and 95 (inclusive) will receive bonus points.