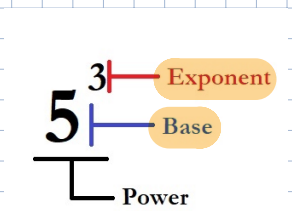
Introduction to Exponents



An exponent is a shorthand way to express the repeated multiplication of a base number.

Definition: If a is a real number, and n is a positive integer, then a^n = a × a × a × a × a ... × a (n times)

- Ex: 3^4 = 3 × 3 × 3 × 3 = 81

However, exponents are not always positive. We also deal with zero, negative, and fractional exponents: a^0 = 1, and a^(-n) = 1 ÷ a^n.

Think of fractional exponents as a combination of powers and roots. The denominator of the fraction tells you which root to take, and the numerator tells you what exponent to raise it to.

- Ex: 8^(1 ÷ 3) = 2

Laws of Exponents

Product Rule: a^m × a^n = a^(m + n)

- Ex: 2^3 × 2^4 = 2^7 = 128

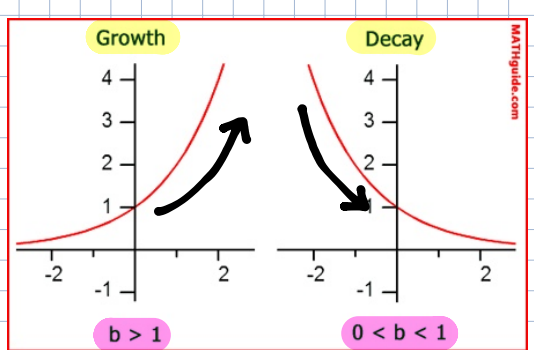
Quotient Rule: a^m ÷ a^n = a^(m - n)

- Ex: 5^4 ÷ 5^(-2) = 5^6 = 15,625

Power Rule: (a^m)^n = a^(m × n)

- Ex: (2^2)^(-3) = 2^(-6) = 1 ÷ 2^6 = 1/64

Exponential Functions



Exponential Growth: f(x) = a(b)^x, where b > 1

- Curves upwards rapidly as x increases
- Ex: f(x) = 2(3)^x

Exponential Decay: f(x) = a(b)^x, where 0 < b < 1

- Curves downwards rapidly as x increases
- Ex: f(x) = 5(0.5) ^ x

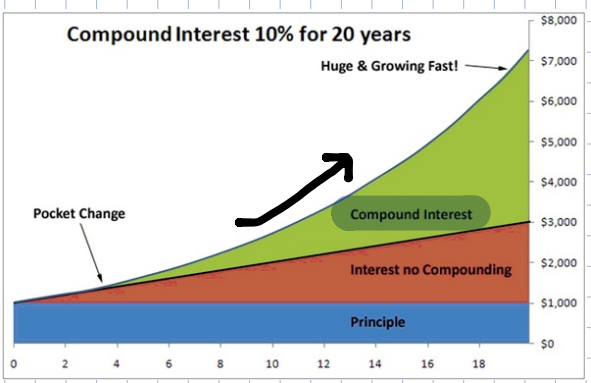
Functions will never reach 0 but will get infinitely close

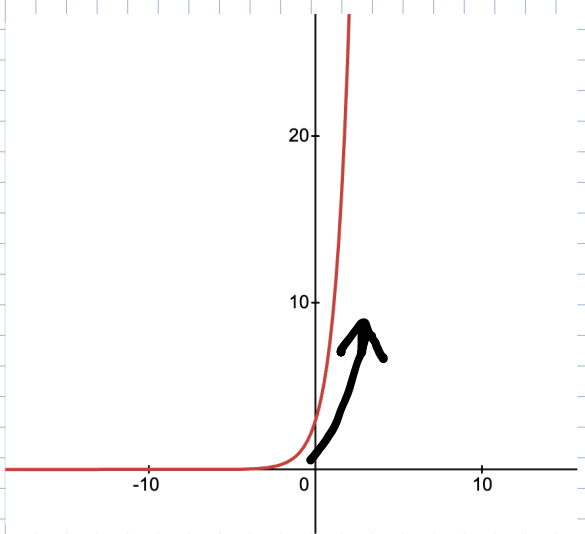
Applications of Exponential Functions

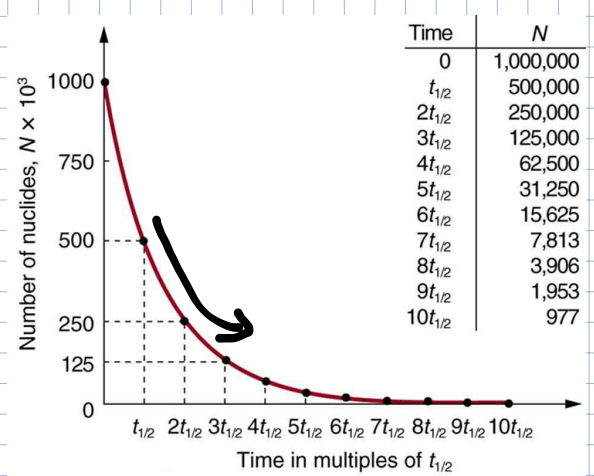
Population Growth: P(t) = p(e ^ (rt)), where p is the initial population, r is the growth rate, and t is time.

Radioactive Decay: A = a(e ^ (-kt)), where a is the initial amount, k is the decay constant, and t is time.

Compound Interest: A = p(1 + r/n) ^ (nt), where p is the principal, r is the interest rate, n is the number of times compounded per year, and t is time in years.







Time	N
0	1,000,000
$t_{1/2}$	500,000
$2t_{1/2}$	250,000
$3t_{1/2}$	125,000
$4t_{1/2}$	62,500
$5t_{1/2}$	31,250
$6t_{1/2}$	15,625
$7t_{1/2}$	7,813
$8t_{1/2}$	3,906
$9t_{1/2}$	1,953
$10t_{1/2}$	977

Summary

- Exponents follow specific rules (product, quotient, power, zero, negative, fractional).
- Exponential functions model rapid growth or decay.
- Applications include finance, population studies, and radioactive decay.